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COMPREHENSIVE EXAMINATION

2019

XI - MATHEMATICS

FROM THE DESK OF ENGR. SADIQ

Engr. Sadiq

SECTION 'B'

Engr. Sadiq

NOTE: Attempt 10 Questions from this section. Do atleast 3 part questions from each part.

COMPLEX NO., ALGEBRA & MATRICES

(1) Find the real and imaginary parts of:

$$\frac{(2+i)^2}{(3-4i)} \quad \text{or} \quad \frac{4-3i}{-1+2i} \quad \text{or} \quad \frac{\sqrt{2}+i}{\sqrt{2}-i}$$

or $i(3+2i)$.

(2)

Or

Given $z_1 = 4+3i$; $z_2 = -1+2i$

Evaluate :

(i) $\frac{z_1}{z_2}$

(ii) $|5z_1 - 4z_2|$

(iii) $\overline{z_1 \times z_2} = \overline{z_1} \times \overline{z_2}$

Or Find the Additive & Multiplicative inverse of $(\sqrt{3}, \sqrt{2})$.

Or Solve the complex equation

$(x+2yi)^2 = xi$ Or

Exercise : $(x, y) \cdot (2, -4) = \left(\frac{28}{3}, \frac{19}{3}\right)$

(2) By using properties of determinants, evaluate/prove;

$$\begin{vmatrix} a+b+2c & b & a \\ c & c+a+2b & a \\ c & b & b+c+2a \end{vmatrix} = 2(a+b+c)^3$$

(3)

OR

$$2(x^3+y^3) = \begin{vmatrix} x+y & x & y \\ y & x+y & x \\ x & y & x+y \end{vmatrix}$$

OR

$$\begin{vmatrix} 1 & x & yz \\ 1 & y & zx \\ 1 & z & xy \end{vmatrix} =$$

$$\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix}$$

OR

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ bc & ca & ab \end{vmatrix} = (a-b)(b-c)(c-a)$$

OR

$$\begin{vmatrix} a+x & a & a \\ a & a+x & a \\ a & a & a+x \end{vmatrix} = a^2(a+9)$$ OR $$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ bc & ca & ab \end{vmatrix} =$$

also... solve

OR

$$\begin{vmatrix} a+x & a & a \\ a & a+x & a \\ a & a & a+x \end{vmatrix} = x^2(3a+x)$$

(3) Solve the system of equations: (& check)

$y^2 + 2z^2 = 17$ OR $x^2 + y^2 = 5$

$y + z = 5$ $(4x-3y)(x-y5) = 0$

OR $3x^2 - y^2 = 12$

$3x^2 + 4y^2 = 192$

(4)

OR $x^2 + y^2 = 5 \; ; \; xy = -2$

(4) Find a matrix X; so that:

$$\begin{bmatrix} 2 & -3 \\ 0 & 1 \end{bmatrix} \cdot X = \begin{bmatrix} -2 & 5 \\ 8 & -7 \end{bmatrix}$$

OR

Prove that:

$$\begin{bmatrix} \sin\theta & -\cos\theta \\ \cos\theta & \sin\theta \end{bmatrix} \begin{bmatrix} \sin\theta & \cos\theta \\ -\cos\theta & \sin\theta \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$ OR

$$\begin{bmatrix} i & 0 & -i \\ 0 & -i & i \\ 0 & i & 0 \end{bmatrix} \begin{bmatrix} -i & i \\ 0 & 0 \\ i & -i \end{bmatrix} = \begin{bmatrix} 2 & -2 \\ -1 & 1 \\ -1 & 1 \end{bmatrix}$$

OR Let $A = \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 7 \\ 9 & 8 & 6 \end{bmatrix}$; verify that

(i) $A^2 + A = (A + I)A$ (ii) $|\text{Adj of } A| = |A|$

OR Solve for x;

$$\begin{bmatrix} -2 & 3 \\ 4 & -1 \end{bmatrix} \begin{bmatrix} 1 & x & 5 \\ 2 & 4 & x \end{bmatrix} \begin{bmatrix} -3 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ -14 \end{bmatrix}$$

Find x; when matrix is singular;

$$\begin{bmatrix} x & 1 & 0 \\ 1 & x & 2 \\ 0 & 2 & x \end{bmatrix}$$

(5)

(5) Prove that the roots of the equation $2cx^2 - 2(a+b+c)x + (a+b) = 0$ are real ; $\forall\, a, b, c \in R$.

OR

Prove that the roots of the equation $x^2 - 2x\left(m + \frac{1}{m}\right) + 3 = 0$ are real ; $\forall\, m \in R$.

OR

Determine the nature of the roots of equation; $x^2 + 16 = 8x$

or

Find all the cube root of $216/-64$ and show that their sum is zero.

or

Show that; $w^{135} + w^{247} + i^{360} = 0$

where w is complex cube root of unity and $i = \sqrt{-1}$.

or

Prove that; $\sqrt{\frac{\alpha}{\beta}} + \sqrt{\frac{\beta}{\alpha}} + \sqrt{\frac{\gamma}{\gamma}} = 0$

where α, β are cube roots of unity.

(b)

Or Prove that;

$$\left(\frac{-1+\sqrt{-3}}{2}\right)^7 + \left(\frac{-1-\sqrt{3}}{2}\right)^7 + I = 0$$

Or $(2+w^2) = \frac{3}{(2+w)}$

Or If α, β are the roots of eqn $ax^2+bx+c=0$; Form an equation whose roots are;

* $\alpha+\frac{1}{\alpha}$ & $\beta+\frac{1}{\beta}$ * $(\alpha+\beta)^2$ & $(\alpha-\beta)^2$

* $\frac{1}{\alpha^2}, \frac{1}{\beta^2}$ * $3+\alpha$ & $3+\beta$

Or For what value of K will $x+5$ be a factor of $2x^3+Kx^2+(-2x)+15$ (By synthetic division)

(b) Solve; $(x+6)(x+1)(x+3)(x-2)+56=0$

Or $\frac{(x-3)}{(x+3)} + \frac{(x+3)}{(x-3)} = 2\frac{4}{15}$

Or $x^4+x^3-4x^2+x+1=0$;

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OR $\sqrt{3x^2+1} = 5 - \frac{4}{\sqrt{3x^2+1}}$

OR $5y^{\frac{1}{2}} + y^{\frac{1}{4}} - 22 = 0$

OR; $\left(t + \frac{1}{t}\right)^2 = 4\left(t - \frac{1}{t}\right)$

"GROUPS, SEQUENCE & SERIES; COUNTING PROBLEMS

(1) Let $*$ be defined in $\mathbb{Z}$; the set of all integers; as $a * b = a + b + 3$. Show that;

a) $*$ is commutative & associative

b) Identity w.r.t $*$ exists in $\mathbb{Z}$

c) Every element of $\mathbb{Z}$ has inverse w.r.t $*$

OR

Let $S = \{1, w, w^2\}$; Construct a composition table w.r.t multiplication in $\mathbb{C}$ and show that:

(8)

(a) Associative law holds in S

(b) 1 is identity element in S

(c) Each element in S has its inverse in S

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Ex

Show that multiplication is a binary operation on $S=\{1,-1\}$ but not on $T=\{1,2\}$.

Ex

Let $S=\{a,b,c,d\}$ where; $A=\{a\}$. $B=\{a,b\}$; $C=\{a,b,c\}$; $\Delta=\emptyset$

Construct multiplication table & show that $\cup$ is a binary operation on S.

Ex

Define a binary operation; $a*b=4ab$; show that $*$ is commutative & $*$ is associative

Ex

Find n & r; if $P_r^n = P_{r+1}^n$ and $^nC_r = {}^nC_{r-1}$

In how many ways can a party of 5 students and 2 teachers be formed out of 15 students and 4 teachers?

Or

A die is rolled twice. What is the probability that the sum of the points on them is 7?

Or

A fair coin is tossed three times; find the probability of getting (i) atleast one head (ii) atmost two heads (iii) two tails (iv) atmost one tail.

Or

In how many ways can 3 books of Mathematics, 2 books of physics and 2 books of chemistry be placed on a shelf so that the books

(10)

of the same subject always remain together.

OR

In how many ways can the letters of the word INFINITE OR INTELLIGENCE / PAKISTAN be arranged.

OR

A word consist of 5 consonants and 4 vowels. Three letters are choosen at random. What is the probability that one vowels will be selected.

(2) Prove by principle of mathematical induction;

$1+3+5+\cdots\cdots+(2n-1)=n^2 ; \forall n \in N$

OR

$a+ar+ar^2+\cdots\cdots+ar^{n-1}=\frac{a(1-r^n)}{(1-r)}$

OR

$4+8+12+\cdots\cdots+4n=2n(n+1)$

(11)

OR $\frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \cdots + \frac{1}{n(n+1)} = \frac{n}{(n+1)}$

OR

$1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$

OR

$2 + 5 + 8 + \cdots + (3n-1) = \frac{n}{2}(3n+1)$; $\forall n \in \mathbb{N}$

OR

$10^{2n} + 3 \cdot 4^{n+2} + 5$ is divisible by 9; $\forall n \in \mathbb{N}$

OR

$a^{2n} - b^{2n}$ is divisible by $a-b$; $\forall n \in \mathbb{N}$.

(3) In the binomial expansion of; $\left(\frac{x}{y} - \frac{y}{x}\right)^{11}$; find the two middle terms; OR $\sqrt{e} = 1 + \frac{1}{2^1} + \frac{1\cdot 3}{2!\,2^2} + \cdots$

OR

Find the middle term in the expansion of $\left(\frac{a}{y} - \frac{y}{a}\right)^{12}$

OR

Expand to four terms; $(1-x^2)^{-1/3}$; $|x| < 1$

or Find the first negative term in the expansion of $\left(1+\frac{3x}{2}\right)^{9/2}$ or

Find the term independent of x in the binomial expansion of: $\left(\frac{4x^2}{3}-\frac{3}{2x}\right)^9$.

(4) Prove that $5^{\frac{1}{2}} \cdot 5^{\frac{1}{4}} \cdot 5^{\frac{1}{8}} \cdots = 5$

or

Find the sum of all natural numbers between 1 & 100 which are exactly divisible by 7.

or Is 83 a term of a sequence.

or

Express the value of $0.42\dot{3}$ as a common Fraction.

or

Find the fifth term from the end of G.P; 2, 6, 8, ---- 4374

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Or. Which term of the sequence 18, 12, 8. ..... is $\frac{512}{729}$.

Or Find the value of n so that $\frac{a^{n+1} + b^{n+1}}{a^n + b^n}$ may become A.M or H.M between a & b.

Or If the n^{th} term of G.P ; 1, 2, 4, 8 ..... is same as that of 256, 128, 64 ..... ; find n.

Or Which term of the H.P ; 6, 2, $\frac{6}{5}$, ... is equal to $\frac{-2}{33}$.

Or Find the sum of seventeen terms of an A.P whose middle term is 5.

Or In a G.P; the First term is 7, the last term is 448 and the sum to n terms is 889. Find the common ratio & no. of terms.

Or

Prove that a, b, c are in A.P; G.P, & H.P according to;

$$\frac{a-b}{b-c} = \frac{a}{a} \quad \text{or} \quad \frac{a}{b} \quad \text{or} \quad \frac{a}{c}$$

Or

If the p^{th} term of an H.P is q; the q^{th} term is p; find the $(p+q)^{th}$ term of an H.P

Or

Find the G.P in which;

$$T_3 = 10 \; ; \quad T_5 = 2\frac{1}{2}$$

Or The sum of first 6 terms of an G.P is 9 times the sum of first 3 terms; Find the common ratio.

(13)

Or Show that product of 'n' G.M's b/w a & b is $(ab)^{n/2}$.

Or If in a G.P $(j+k)^{th}$ term is x and $(j-k)^{th}$ term is y; prove that j^{th} term is $\frac{y}{\sqrt{xy}}$.

Or The sum of First n terms of two A.P's are in ratio $3n+31 : 5n-3$; show that their 9^{th} terms are equal.

Or For what values of 'x' do the numbers $1, x^2, 6-x^2$ taken in that order to form a G.P.

Or Find the sum of the First 'n' terms of the following series;

$6 + 66 + 666 + \cdots\cdots$

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TRIGONOMETRY

(1) Draw the graph of $\sin(-\theta)$ where $0 \leq \theta \leq 2\pi$.

Or Find the period of $\sin\frac{x}{2} + \cot\frac{x}{4}$.

(2) Prove that $\tan^{-1}\frac{1}{3} + \frac{1}{2}\tan^{-1}\frac{1}{7} = \frac{\pi}{8}$

Or $\tan^{-1}\theta = \sin^{-1}\frac{\theta}{\sqrt{1+\theta^2}}$

Or $\sin^{-1}\theta + \cos^{-1}\theta = \frac{\pi}{2}$

(13) If the point on the rim of a 21 cm diameter fly wheel travels 5040 m in a minute; through how many radians does the fly wheel turn in one second?

OR

A car is running on a circular track of a radius

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equal to half of the arc of the circle travelled by a car. Find the angle subtended by the arc at the center of the circular path.

or

A belt 36 m long passes around a 1.6 cm diameter pulley. As the belt makes two complete revolutions in a minute. How many radians does the wheel turn in one second.

(9) If $\cot\theta = 3$ and $\sin\theta$ is +ve, Find the remaining trigonometric ratios or if $\sin\theta = \frac{\sqrt{3}}{2}$ and $P(\theta)$ is in the 2nd quadrant find remaining trigonometric ratio. Using defination of $x^2 + y^2 = 1$

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OR Prove that: $r_1 r_2 r_3 = r s^2$

or $\Delta = 4Rr \cos\frac{\alpha}{2} \cos\frac{\beta}{2} \cos\frac{\gamma}{2}$

or $\Delta = r^2 \cot\frac{\alpha}{2} \cot\frac{\beta}{2} \cot\frac{\gamma}{2}$

or $r_2 = b \dfrac{\cos\frac{\alpha}{2} \cos\frac{\gamma}{2}}{\cos\frac{\beta}{2}}$

OR Solve the equation;

$\sin\theta + \cos\theta = 1$ or $1 - \cos\theta = \frac{1}{2}$

or $\sqrt{1+\cos\theta} - \sqrt{1-\sin\theta} = 1$

OR

$12\sin^2\theta - 4\sin\theta - 1 = 0$

OR

$\tan\theta \cot 2\theta = 3$

OR

$\sqrt{3}\cos\theta + \sin\theta - 2 = 0$

OR

$\cos 2\theta + 1 = \cos\theta$

OR If $a = b = c$; then prove that

$r_1 : R : r = 3 : 2 : 1$

(5) Prove that for a triangle;

$$\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} = \frac{1}{r}$$

OR

$$r_1 = (s-c)\cot\frac{\beta}{2} = 4R\sin\frac{\alpha}{2}\cos\frac{\beta}{2}\cos\frac{\gamma}{2}$$

OR Show that (2,1); (5,1); (2,6) are the vertices of a right angled triangle.

OR

Find the area of triangle ΔABC; if $\alpha = 30°$; $\beta = 63°$ $c = 7.3$ cm.

SECTION "C"

Q(a) Show that the sum of $(p+q)^{th}$ & $(p-q)^{th}$ term of an A.P is equal to the twice of p^{th} term.

OR

If a rubber ball that is dropped on the floor from a height 27 m always rebounds one-third of the distance of the previous fall. Find the distance it will have travelled before hitting the ground for the ninth time.

OR

The sum of four numbers in A.P is 4 and the sum of their squares is 24. Find the numbers.

OR

Find three numbers whose sum is 19 and sum of their squares is 133 and are forming G.P.

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Q. The sum of 20 terms of an A.P whose 4th term is 7 and 7th term is 13.

OR

Insert four harmonic means b/w 7 and 1/6.

Q. If the roots of the equation $a(b-c)x^2 + b(c-a)x + c(a-b) = 0$ are equal; prove that a, b & c are in H.P

Q. The sum of infinite terms of a G.P is 4 and sum of their cubes is 192. find the G.P.

Q. If $\frac{1+3+5+\cdots\cdots+n \text{ terms}}{2+4+6+\cdots\cdots+n \text{ terms}} = 0.95$

find 'n'

(1) (b) A man is standing on the bank of a river. He observes that the measure of the angle subtended by a tree on the opposite bank is 65°. When he retreats 35 meters from the bank, then he find measure of angle is to be 35°. Find the height of the tree and the width of the river.

Or

If one side of a triangle building is y-units. in length. another side is 3 times as long as the first one, and the angle subtended b/w two sides measures 35°. Find the other two angles and length of third side.

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Or If the length of the larger side of llgm is 65 cm and one of its diagonals make angles of measure 20° and 45° with a pair of adjacent sides; find the length of diagonal.

Or Two hikers starts from rest on the same point; one walks 9 Km heading east, the other one 10 Km heading 55° north east. How far apart are they at the end of their walks.

Or

Three points A, B, C form a triangle such that the ratio of the measures of their angles is 1:2:3 Find

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the ratio of length of their sides.

OR

A piece of plastic strip 1 meter long is bent to form an isosceles triangle with 95° as measure of its largest angle. Find the length of sides.

(7) Use the Adjoint method to solve the given equations;

$$2x - y + 2z = 4$$
$$x + 10y - 3z = 10$$
$$x - y - z = 6$$

OR

Find the inverse of;

$$A = \begin{bmatrix} 1 & P & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ by Adjoint method & also prove that;

$$A \cdot A^{-1} = I$$

(25)

(3) Prove that:-

* $\tan\frac{\theta}{2} = \frac{1-\cos\theta}{\sin\theta}$

* $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

* $\cos 36^\circ = \frac{\sin 9^\circ - \cos 9^\circ}{\cos 9^\circ - \sin 9^\circ}$

* $\frac{\tan\left(\frac{\theta+\phi}{2}\right)}{\tan\left(\frac{\theta-\phi}{2}\right)} = \frac{\sin\theta + \sin\phi}{\sin\theta - \sin\phi}$

* $\frac{1+\sec\theta}{1-\sec\theta} = \frac{\tan\theta + \sin\theta}{\sin\theta - \tan\theta}$

* $\frac{\sin 3\theta}{\sin\theta} - \frac{\cos 3\theta}{\cos\theta} = 2$

* $\cos^4\theta - \sin^4\theta = 1 - 2\sin^2\theta$

* $\frac{\cot\theta + \operatorname{cosec}\theta}{\sin\theta + \tan\theta} = \operatorname{cosec}\theta\cot\theta$

* $\frac{\sin 7\theta - \sin 5\theta}{\cos 7\theta + \cos 5\theta} = \tan\theta$

* $\frac{\sin(\theta+\phi)}{\cos\theta\cos\phi} = \tan\theta + \tan\phi$

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(4) Prove that;

$$\sin\frac{\alpha}{2} = \sqrt{\frac{(s-b)(s-c)}{bc}}$$

or $r_2 = \frac{\Delta}{s-b}$ or $r = \frac{\Delta}{s}$

or

Derive the law of Tangents / cosine law.

(5) Prove that;

$$\sqrt{\frac{y}{y+x}} + \sqrt{\frac{y}{y-x}} = 2 + \frac{3x^2}{4y^2}$$

or

If ; $y = 2x + 3x^2 + 4x^3 + \cdots$

Show that;

$$x = \frac{y}{2} - \frac{1\cdot 3}{2^2 2!}y^2 + \cdots\cdots$$

or If $y = \frac{3}{4} + \frac{3\cdot 5}{4\cdot 3} + \frac{3\cdot 5\cdot 7}{4\cdot 3\cdot 2} + \cdots$

then prove that,

$$y^2 - 2y + (-7) = 0$$

or Prove that $(1+x+x^2+\cdots)(1+2x+3x^2+\cdots) = \frac{1}{2}(1\cdot 2 + 2\cdot 3x + \cdots)$