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SUPPLEMENTARY EXAMINATION 2019 (TARGET PAPER)
MATHEMATICS PAPER - II
(Science Pre-Engineering & General Science Group)

IMPORTANT INSTRUCTIONS:

This Paper consisting of Multiple Choice Questions (Section 'A') and all of them to be answered. Its total duration is 20 minutes only.

SECTION 'A' (MULTIPLE– CHOICE QUESTIONS) (Marks: 20)

Q.1: *Choose the correct answer for each from the given options.*

1. If e = 1 ; then the conic is:
   *Circle *Parabola *Ellipse *Rectangular Hyperbola
2. $\int \frac{dx}{a^2 + x^2}$:
   *$\tan^{-1}\frac{x}{a}$ + C *$\frac{1}{a}\tan^{-1}\frac{x}{a}$ + C *$\frac{1}{a}\tan^{-1}\frac{a}{x}$ + C *$a\tan^{-1}x$+ C
3. If f(*x*) = *sinx* and *g*(*y*) = $\sin^{-1} y$ then *fog* is:
   *x *y *$\sin^{-1}(\sin x)$ *$\sin(\sin^{-1} x)$
4. Which of the following circle passes through the origin?
   *$x^2 + y^2 + 8x + 7 = 0$ *$x^2 + y^2 - 9x + 11 = 0$ *$x^2 + y^2 + 8x + 11y = 0$ *$x^2 + y^2 - 8x + 11y + 19 = 0$
5. The n^{th} term of the sequence 3, 7, 11, 15…… is:
   **4n-3* **4n-1* **4n+1* **4n+3*
6. If the straight line is parallel to y-axis, then its slope is:
   *1 *0 *-1 *∞
7. $If\ b^2 = a^2(e^2 - 1); then\ the\ conic\ is$:
   *Circle *Parabola *Hyperbola *Ellipse
8. If $\vec{A}$ and B are any two vectors such that $\vec{A} \cdot \vec{B} = 0$ then the vectors are:
   **Parallel* **Perpendicular* **Neither parallel nor perpendicular* **Coincident*
9. $\lim_{n\to\infty}\left(1 + \frac{1}{n}\right)^n$ =:
   *0 *∞ *1 *e
10. The length of major axis of ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ is:
    *6 *4 *13 *9
11. The centre of the circle $x^2 + y^2 + 6x - 10y + 33 = 0$ is:
    *(-3, 5) *(-3, -5) *(3, -5) *(3, 5)
12. $\lim_{x\to a}\frac{f(x) - f(a)}{x - a}$ *is equal to*:
    *$f'(x)$ *$f'(a)$ *$f'(0)$ *$f'(1)$
13. The vertex of the parabola $(x - 1)^2 = 8(y + 2)$ *is*:
    *(1, -2) *(0, 1) *(2, 0) *(0, 0)
14. If $f(x) = \tan^{-1} 2x$; *then* $f'(x)$ *is*:
    *$\frac{1}{1 + x^2}$ *$\frac{1}{4 + x^2}$ *$\frac{1}{1 + 4x^2}$ *$\frac{2}{1 + 4x^2}$
15. The distance of a point (1, 1) from the line *x – y + 3 = 0 is*:
    *0 *$\frac{5}{\sqrt{2}}$ *units* *$\frac{5}{\sqrt{2}}$ *units* *3 *units*
16. A function $f(x) = \frac{x}{|x|}; x \neq 0$ *is*:
    *even function *odd function *circular function *neither even nor odd
17. The distance between the foci of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$:
    *2a *2c *2b *2e
18. $\int e^{tanx} \sec^2 x\ dx$ *is equal to*:
    *$e^{tanx} + C$ *$\sec^2 x + C$ **Tanx* + *C* *$e^{secx} + C$
19. The slope of the tangent to the curve *y* = 6x^2 at (1, -1) is:
    *-12 *12 *15 *6
20. The point of intersection of the altitudes of a triangle is called:
    *Orthocentre *Circum-centre *Centroid *Incentre

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IMPORTANT INSTRUCTIONS:

The use of calculator is allowed. All notations are used in their usual meanings.

SECTION 'B'
(SHORT–ANSWER QUESTIONS)

Marks: 50

NOTE : Answer any **TEN** Part questions from this Section, selecting atleast **THREE** Part questions from each question. All questions carry equal marks.

ANALYTIC GEOMETRY (STRAIGHT LINE) & VECTOR ALGEBRA

1. (i) Find the angle between the lines represented by $x^2 + 2xy - 3y^2$ **OR** Prove that the diagonals of an isosceles trapezoid are equal? **OR** Show that the points (2, 4), (3, -4) and (3, 4) are not collinear. Find the area of the triangle formed by these points. **OR** The angle from the line through (2,7) and (-6,5) to a line through (1, -4) is 135^0. Find the equation of the second line.
OR Find the equation of a line which is perpendicular to 2x + 3y + 4 = 0 and passes through the point (2, 1).

(ii) Show that the equation of line through origin making an angle of measure $\emptyset$ with the line $y = mx + b$ is $\frac{y}{x} = \frac{m + Tan\emptyset}{1 - mTan\emptyset}$ **OR** Find the value of **k** for which the two lines (k – 1)x + ky – 5=0, kx + (2k – 1)y + 7=0 intersect at a point lying on the axis of x. **OR** The gradient of one of the lines $ax^2 + 2hxy + by^2 = 0$ is five times that of the other, show that $5h^2 = 9ab$ **OR** Find the centroid of the triangle; the equations of whose sides are $12x^2 - 20xy + 7y^2 = 0$ and $2x - 3y + 4 = 0$. **OR** Find in what ratios the join of (-2, 2) and (4, 5) is cut by the axes of coordinates. **OR** Find the equation of line *AB* such that 'A' is the mid point of (-4, 4) and (2, 2) and 'B' is three-fifth the way from (5, 3) and (-3, -2). **OR** The coordinates of the vertices of the triangle are A (10, 4), B (-4, 9) and C (-2, -1). Find the equation of median through 'A'.

(iii) A straight line passes through the points A(2, -3) and B(-6, 5). On this line, find a point whose ordinate is equal to -5. **OR** Determine the equation of a line which passes through the point (-2, -4) and has sum of its intercepts equal to 3. **OR** Find the equation of the line passes through the intersection of lines 2x + 3y + 1 = 0, 3x – 4y – 5 = 0 and passing through the point (2, 1) **OR** Find the equation of the perpendicular bisector of the line joining the points A(15, 14) and B(-3, -4). **OR** For what value of 'a' and 'b' for which the line; (a + 2b – 3)x + (2a – b + 1)y + 6a + 9 = 0 is parallel to axis of X and the Y-Intercept is -3. Also write the equation of this line.

(iv) Find the unit vector perpendicular to $\vec{a} = i - 3j + 2k$; $\vec{b} = -3i + 2k$ **OR**
Evaluate Scalar Triple Product; $[\hat{k}, 3\hat{\imath} + 2\hat{\jmath} - 2\hat{k}, \hat{\imath} + \hat{\jmath}]$
OR Find the volume of the parallelepiped, whose three edges are represented by $\vec{a} = j$; $\vec{b} = -i + 2j$ *and* $\vec{c} = -i - j + 3k$ **OR** Prove that; $[\vec{a},\ 2\vec{b} - 3\vec{c}, -2\vec{a} + \vec{b} + \vec{c}] = 5[\vec{a}, \vec{b}, \vec{c}\,]$

(v) Find the constant 'a' such that the vectors $2\hat{\imath} - \hat{\jmath} + \hat{k}$ and $3\hat{\imath} + a\hat{\jmath} + 5\hat{k}$ are coplanar. **OR** Find $\cos(\overrightarrow{AB}, \overrightarrow{AC})$ *in a triangle*; whose vertices are A(-2, 0), B(4, 3) and C(5, -1) **OR** Find the angle between positive Y-axis and the vector $\vec{a} = -i + 3j + 2k$

ANALYTIC GEOMETRY (CONIC SECTIONS)

2. (i) Find the equation of circles touching each axis at a distance of 5 units from the origin. **OR** Find the equation of the circle touching x-axis and passes through the points (1, -2) and (3, -4) **OR** Prove that the straight line y = x + c$\sqrt{2}$ touches the circle $x^2 + y^2 = c^2$, find its point of contact. **OR** Prove that the product of abscissa of the points where the straight line $y = mx$ meets the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ is equal to $\frac{c}{1 + m^2}$

3. (ii) Find the equation of a circle concentric with the circle; $x^2 + y^2 - 4x - 6y - 23 = 0$ and passing through the point (-2, -4). **OR** Find the equation of circle whose diameter is the latus rectum of the parabola $x^2 = -36y$
OR Find the equation of circle containing the points (-1, -1) and (3, 1) and with the center on the line x – y + 10 = 0
OR Find the equation of the circle containing the point (6, 0) and touching the line x = y at the point (4, 4).

(iii) Show that the eccentricities e_1 and e_2 of two conjugate hyperbolas satisfy the relation;

$$e_1^2 + e_2^2 = e_1^2\, e_2^2$$ **OR**

Determine the vertex, focus and equation of directrix of the curve $x^2 + 4x + 4y - 12 = 0$ **OR**
Find the equation of an ellipse having centre (-2, 3), end points of minor axis (0, ±3) and length of latus rectum 3.
OR Find the equation of an ellipse whose centre is at the origin and e = $\frac{2}{3}$, length of latus rectum = $\frac{20}{3}$ and minor axis is along the x-axis. **OR** Find the distance between the directrices of the hyperbola $16x^2 - 9y^2 = 144$
(iv) Find the equation of parabola with the focus at (-5, 3) and directrix y = 7. **OR** Find the equation of common tangents to parabola $y^2 = 4x$ and the rectangular hyperbola $y^2 - x^2 = 4$ **OR** Find the coordinates of the centre, foci, eccentricity, directrices and length of latus rectum of the hyperbola $\frac{(x+2)^2}{4} - \frac{(y-1)^2}{9} = 1$
OR Find the eccentricity, foci and equations of directrices of $25x^2 + 9y^2 = 225$ **OR** Prove that the curves $3x^2 - y^2 = 12$ and $x^2 + 3y^2 = 24$ intersect at right angle at the point $(\sqrt{6}, \sqrt{6})$ **OR**

If (x_1, y_1) *and* (x_2, y_2) are the coordinates of the extremities of focal chord of the parabola $y^2 = 4cx$.

Prove that; $x_1.x_2 = c^2$ and $y_1.y_2 = -4c^2$

(v) Find the eccentricity of Hyperbola whose Latus rectum is four times as that of the transverse axis. **OR**
Find the equation of circle passing through the focus of parabola $x^2 + 8y = 0$ and foci of ellipse

$$16x^2 + 25y^2 = 400$$

OR The length of major axis of an ellipse is 25 units and its foci are the points (±5, 0). Find its equation.
OR Find the equation of an ellipse with centre at origin satisfying the conditions e = $\frac{2}{3}$ and directrix $x - 3 = 0$
OR Find the equations of tangents at the ends of the latus rectum of the parabola $y^2 = 4ax$.

CALCULUS

4. (i) Find the derivative by the first principle, at *x* = *a* ∈ D (*f*), of the function $f(x) = \ cosx$ **OR** $f(x) = (x)^{\frac{2}{3}}$
OR $f(x) = 2\sin 2x$ **OR** $f(x) = sec\,x$ **OR** $f(x) = \tan x$ **OR** $f(x) = 3x^2 - x$
(ii) Evaluate ANY TWO of the following:

(a) $\lim\limits_{x \to a} \frac{x^m - a^m}{x^n - a^n}$ **OR** $\lim\limits_{x \to 0} \frac{e^{mx} - e^{nx}}{x}$ **OR** $\lim\limits_{x \to \infty} \left(1 + \frac{2}{x}\right)^x$ (b) $\lim\limits_{x \to 3} \frac{x^2 - 9}{x - 3}$ **OR** $\lim\limits_{x \to 1} \frac{1}{1 - x} - \frac{3}{1 - x^3}$

(c) $\lim\limits_{x \to 0} \frac{\sqrt{1 + x} - \sqrt{1 - x}}{x}$ **OR** $\lim\limits_{x \to 0} \frac{3e^x - e^{-x} - 2}{x}$ **OR** $\lim\limits_{x \to 0} \frac{\csc x - \cot x}{x}$ (d) $\lim\limits_{x \to 0} \frac{1 - \cos 5x}{1 - \cos 7x}$ **OR** $\lim\limits_{x \to 0} \frac{x}{\sqrt{1 - \cos x}}$

(e) $\lim\limits_{x \to 0} \frac{1 - \cos x}{\sin x}$ **OR** $\lim\limits_{x \to 0} \frac{\sin^2 \frac{x}{3}}{x^2}$ **OR** $\lim\limits_{x \to \infty} \frac{2t^2 - 2t + 1}{t^2 + 4}$ **OR** $\lim\limits_{x \to 0} \frac{\sqrt{x + a} - \sqrt{a}}{x}$

(iii) Two polynomial functions f, g are defined by $f(x) = x^2 - 3x + 4;\ g(x) = x + 1;\ \forall\, x \in R$; Find the composite functions *fog*, *gof* and show that; $fog \neq gof$. **OR**

Find the nth term and the limit of the sequence: $\frac{1.2}{3.4}, \frac{3.4}{5.6}, \frac{5.6}{7.8}, \dots\dots\dots$ **OR**

A function $f: R \to R$ is defined by $f(x) = \begin{cases} 1 - x, & \forall\, x \in (-\infty, 1) \\ 1 + x, & \forall\, x \in [1, 2] \\ 1 & \forall\, x \in (2, +\infty) \end{cases}$

Find; (i) The image of zero (ii) The value of *f* at 3 (iii) $f(\sqrt{3})$ (iv) *f(1)* (v) The image of 5

(iv) Solve the differential equation; $y\frac{dy}{dx} = 3x^2$; y(2) = 1 *OR* $2 + 2y\frac{dy}{dx} = 1 + 3x^2;\ \ y(2) = 1$

OR $y(1 + x^2)\frac{dy}{dx} = x(1 + y^2)^2$ *OR* $\frac{dy}{dx} = \frac{\sqrt{1 + \cos y}}{siny}$; $y(3) = \frac{\pi}{2}$ *OR* $y\frac{dy}{dx} = x(y^4 + 2y^2 + 1);\ \ y(-3) = 1$

OR

Find the area above X-axis; under the following:

$y = x^2 - 2x + 5;\ \ x = 0, x = 1$ **OR** $y^2 = x;\ \ x = 1, x = 3$ **OR** $x^2 + y^2 = 25;\ \ a = 3, b = 4$

OR $y = 3x^4 - 2x^3 + 1;\ \ x = 1, x = 2$ **OR** $\frac{x^2}{4} + \frac{y^2}{9} = 1;\ \ x = -1,\ x = 1$

(v) Calculate the Approximate value of sin 46^0 *OR* Tan 44^0 *OR* cos 46^0

OR If $\log_{10} e = 0.4343$; Using differentials calculate $\log_{10}(10.1)$ **OR**

If y = *acosx* + *bsinx*; Show that $\frac{d^2y}{dx^2} + y = 0$

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SECTION 'C'
(DETAILED – ANSWER QUESTIONS) *(Marks: 30)*

NOTE : Attempt any **TWO** questions from this Section Including **Question number 4** which is compulsory.

Q.4. Evaluate **ANY FOUR** of the following:

(i) $\int lnx\, dx$ **OR** $\int xlnx\, dx$ **OR** $\int sin\sqrt{2x}\, dx$ (ii) $\int_0^{\frac{\pi}{3}} \frac{dx}{1-sinx}$ **OR** $\int cose^x . e^x\, dx$

(iii) $\int_0^1 \frac{x^3}{\sqrt{4-x^2}}\, dx$ **OR** $\int \frac{7x-25}{(x-3)(x-4)}$ **OR** $\int \frac{sinx\, dx}{(1+cosx)(2+cosx)}$ (iv) $\int_{-2}^{1} \sqrt{2-x}\, dx$ **OR** $\int cos4x\, cos2x\, dx$

(v) $\int x \tan^{-1} x\, dx$ **OR** $\int 6x^5\, e^{x^3} dx$ **OR** $\int \frac{x^3\, dx}{\sqrt{a^2-x^2}}$ *(Using Trigonometric Substitution)* **OR** $\int \frac{x\, dx}{\sqrt{1+x}}$

(vi) $\int_0^{2\sqrt{3}} \frac{x^2\, dx}{\sqrt{x^2+4}}$ *(Using Trigonometric Substitution)* **OR** $\int \frac{dx}{\sqrt{5+4x-x^2}}$ **OR** $\int (x^2+bx+c)^{\frac{2}{3}} \left(x+\frac{b}{2}\right) dx$

OR $\int_0^{\frac{\pi}{6}} \sin^5 3x\, \cos^3 3x\, dx$ **OR** $\int \frac{(2x-1)\, dx}{x\,(x-1)\,(x-3)}$ **OR** $\int \cos^5 x\, dx$ **OR** $\int_{-2}^{1} \sqrt{2-x}\, dx$

Q.5. (a) Find the incentre of the triangle, the equations of whose sides are x = 3, y = 4 and 4x + 3y = 12.

OR Prove that the angle between the conics $y^2 = 4bx$ and $x^2 = 4ay$ at a point other than origin is $\theta = \tan^{-1}\frac{3}{2}\left(\frac{a^{\frac{1}{3}} b^{\frac{1}{3}}}{a^{\frac{2}{3}}+b^{\frac{2}{3}}}\right)$

OR Show that the lines $x^2 - 4xy + y^2 = 0$ and $x + y = 3$ form an equilateral triangle. Also find the area of the triangle.

(b) Find a right angled triangle of maximum area with hypotenuse of length 'h'. **OR** The line joining the points P (2, -3) and Q (-4, 5) is trisected. Find the coordinates of the points of trisection.

OR A line whose y-intercept is one less than its x-intercept forms a triangle of area 6 square units with the coordinate axes. What is its equation.

Q.6. (a) Prove that the line $lx + my + n = 0$ and the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ have just one point in common if $a^2l^2 + b^2m^2 - n^2 = 0$ **OR** Prove that the two circles $x^2 + y^2 + 2gx + c = 0$ and $x^2 + y^2 + 2fy + c = 0$ touch each other if $\frac{1}{f^2} + \frac{1}{g^2} = \frac{1}{c}$

(b) Find the condition that the conic $ax^2 + by^2 = 1$ and $a'x^2 + b'y^2 = 1$ should cut orthogonally. **OR**

From a square sheet of cardboard with side 6a units is made a topless box of maximum volume by cutting equal squares at the corners and removing them and turning up the sides. Prove that the length of the side of the square is a units.

OR Evaluate $\frac{dy}{dx}$ of **ANY TWO** of the following:

(i) $y = \cot^{-1}\left(\frac{2x}{1-x^2}\right)$ *OR* $y = ln\left(\frac{e^x}{1-e^x}\right)$ (ii) $e^x lny = \sin^{-1} y$ *OR* $y = \frac{1}{2}\tan^2 x + lncosx$

(iii) $x = lnt + sint,\ y = e^t + cost$ *OR* $y = e^{sinx}\, cosx$ (iv) $x = a\sin^2\theta, y = b\sin^2\theta$ at $\theta = \frac{\pi}{2}$

(iv) $\sqrt{x^2+y^2} = ln(x^2 - y^2)$ *OR* $x^y . y^x = 1$ *OR* $y = (x)^{sinx\, +\, cosx}$ *OR* $y = e^{sinx}\, cosx$

OR Find the maximum and minimum values of the function $f(x) = e^x cosx$ *OR* $f(x) = \frac{x}{lnx}$ *OR* $f(x) = -x^4 + 2x^2$ *OR* $f(x) = 2e^x + e^{-x}$ *OR* $f(x) = x^3 - 9x^2 + 15x + 3$

-------------------- **BEST OF LUCK** ------------------------

Marked with **'RED BOLD'** are the '**MOST IMPORTANT**' Questions.

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